P: ISSN NO.: 2394-0344 RNI No.UPBIL/2016/67980 VOL-3* ISSUE-11* February 2019
E: ISSN NO.: 2455-0817 Remarking An Analisation

Flow Through A Permeable Bed Bounded By A Horizontal Stretching Sheet At The Bottom And An Impermeable Plate With Injection At The Top

Abstract

The steady two-dimensional viscous incompressible flow through a naturally permeable bed (Brinkman Model) having an impermeable top and a stretching sheet at the bottom, is examined. The flow is caused by injecting fluid at the top and stretching of the bottom sheet. The effect of the injection, permeability and stretching parameters on the flow behavior is examined.

Keywords: Viscous, Flow and Injecting Fluid.

B.S. Rawat
Assistant Professor,
Deptt.of Mathematics,
D.B.S. (PG) College,
Dehradun, Uttarakhand, India.

Introduction

The study of flow through porous media is of principal interest in many scientific and engineering applications. This flow is represented by a macroscopic law, known as Darcy's law. Brinkman (1947) suggested an empirical modification to it, and Brinkman's equation is one of the most important equations, used to describe flow in porous media. It has become a major tool in the theoretical investigation of flow in porous media, Saffman (1971, 73), Kim and Russel (1985), Rubinstein (1986).

The flow due to a stretching sheet in a porous media has importance in the polymer industry, Mecormack*et al*. (1973), Brady et al. (1981), Wang (1984), etc. have studied the flow caused by stretching sheet, however, the flow in porous media due to a stretching sheet does not seemto have received much attention.

In the present investigation the steady two-dimensional flow of viscous incompressible fluid in a permeable Brinkman bed bounded by a horizontal stretching sheet at the bottom and an impermeable plate with injection at the top, is studied. The results for velocity distribution, stream function, pressure coefficient and skin-friction have been obtained.

Aim of the Study

The main aim of the investigations presented here is to analysecertaingeometries in the flow patterns of non-Newtonian fluids different from Newtonianfluids.The study of boundary layer flow of Newtonian/non-Newtonian (Walters'sliquid B/Power-law fluid) fluids over a stretching sheet has generated much interestover the years because of its numerous industrial applications such as aerodynamicextrusion of polymer sheets, continuous stretching, rolling and manufacturing ofplastic films and artificial fibres. As we know that, in the field of aeronautics, thatsuction or injection can delay the separation and transition from laminar to turbulentmotions in the boundary layer. These ultimately results in the reduction of drag oversolid bodies. Therefore it is of interest to study the suction or blowing on the flows ofnon-Newtonian fluids. Keeping this in view, in the present thesis to find out how thestresses on the bodies are affected by the suction or blowing and know the behavior of non-Newtonian parameter to these stresses in relation to the contribution of usualviscosity.

Review of Literature:

Sakiadis, (1961a,b,c) was the first to study a two-dimensional, axisymmetric boundary layer flow over a stretched surface moving with constant velocity. Both exactand approximate solutions were presented for laminar flow with the latter being obtained by the integral method. Due to the entertainment of the ambient liquid, this situation represents a different class of boundary layer problems (Blasius, (1973)) and has asolution substantially different from that of boundary layer flow over a semi-infinite flatplate. Crane, (1970) pointed out that in the polymer industry it issometimes necessary to consider a stretching plastic sheet. Magyariand Keller, (2000) studied the steady boundary layer flow induced by permeable stretching surfaces with variable temperature distribution under Reynolds' analogy. Reynolds'analogy makes use of the advantage of all the exact analytic solutions of the momentum and energy equations.

Magyari and Keller, (2001) analyzed the free laminar jets ofclassical hydrodynamics that maybe identified with certain boundary–layer flows inducedby continuous surfaces immersed in quiescent in compressible liquids and stretched with well defined velocities.Partha et al. (2005) have examinedthe mixed convection flow and heat transfer from an exponentially stretching vertical surface in a quiescent liquid using a similarity solution.

Hayat et al. (2017) analyzed the convective flow of viscousnanofluid between two rotating disks. Surface drag forcesand temperature gradient for thermally radiative two-phaseflow using Ag–water and Cu–water nanofluids are exploredby Hayat et al.(2018). Second thermodynamics law in engineering context OztopHF(2012)is considered more appropriate than the first law. Internalfriction effects, spinning, vibration and kinetic energy lose useful heat which cannot be transformed into work. In real life these losses of energy cannot be recovered without extra effort. Irrever sibilities effects are very important in a system.

Formulation of the problem

Steady laminar flow of a viscous Incompressible fluid through a permeable bed of finite thickness h, having an impermeable top and a stretching sheet at the bottom, is considered. The fluid motion is caused by injection fluid at the top and stretching of the sheet at the bottom. A cartesian co-ordinate system is used with the origin at the bottom and y-axis normal to it.

The flow in the porous matrix (0 ≤ y ≤ h) is governed by the Brink man's equations:

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial x} - \nu\frac{u}{k} + \nu\left[\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right] \qquad \text{----- (2.1)}$$

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial y} - \nu\frac{v}{k} + \nu\left[\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right] \qquad \text{----- (2.2)}$$

and $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ----- (2.3)

where u, v are the fluid velocity components along the x and y directions in the porous matrix, p the pressure ρ the density, ν the kinematic viscosity and k the permeability of the porous medium.

The boundary conditions are:

$u=C_X$, $v = 0$ at y=0

and $u=0$, $v = -v_0$ at y= h ----- (2.4)

where $u=C_x$, represents the stretching velocity of the sheet with C > 0. The stretching sheet is stretched by introducing two equal and opposite forces so that the position of the point (0, 0) remains unchanged.

Method of Solution:

Let $u = C_x f'(\eta), \quad v = -C_h f(\eta)$

and $\eta = \frac{y}{h}$ ----- (3.1)

where a prime denotes differentiation with respect to η. Substituting (3.1) intoequation (2.1) to (2.4), we have

$$\frac{C_x^2}{R}\left[\frac{1}{\beta}f'(\eta) - f'''(\eta)\right] = -\frac{1}{\rho}\frac{\partial p}{\partial \eta} \qquad \text{----- (3.2)}$$

$$\frac{C_h^2}{R}\left[f''(\eta) - \frac{1}{\beta}f(\eta)\right] = -\frac{1}{\rho h}\frac{\partial p}{\partial \eta} \qquad \text{----- (3.3)}$$

and the corresponding boundary conditions are:

$f'(0) = 1$, $f(0) = 0$, at $\eta = 0$,

and $f'(1) = 0$, $f(1) = \lambda$, at $\eta = 1$, ----- (3.4)

where

$R = \frac{C_h^2}{\nu}$, $\beta = \frac{k}{h^2}$ and $\lambda = \frac{v_0}{C_h}$,

Solutions

The solutions are obtained on solving (3.2) and (3.3) under boundary conditions (3.4). Thus, we have

$$f' = C_1 e^{\frac{\eta}{\sqrt{\beta}}} + C_2 e^{\frac{-\eta}{\sqrt{\beta}}} - A\beta \qquad \text{----- (4.1)}$$

$$f(\eta) = C_1\sqrt{\beta}e^{\frac{\eta}{\sqrt{\beta}}} - C_2\sqrt{\beta}e^{\frac{-\eta}{\sqrt{\beta}}} - A\beta\eta + C_3 \qquad \text{,----- (4.2)}$$

Where $C_1 = \frac{1}{1-e^{\frac{\eta}{\sqrt{\beta}}}} + C_2 e^{\frac{-1}{\sqrt{\beta}}}$

$$C_2 = \frac{\left[\left(1-e^{\frac{1}{\sqrt{\beta}}}\right)(\sqrt{\beta}-\lambda-1)+1\right]}{4\sqrt{\beta}-4\sqrt{\beta}\cosh\left(\frac{1}{\sqrt{\beta}}\right)+2\sinh\left(\frac{1}{\sqrt{\beta}}\right)}$$

$C_3 = \sqrt{\beta}(C_1 - C_2)$

$A = \frac{1}{\beta}(C_1 + C_2 - 1)$ ----- (4.3)

By introducing the following dimensionless quantities:

$\bar{u} = \frac{u}{\nu/h}$, $\bar{v} = \frac{u}{\nu/h}$,$\xi = \frac{x}{h}$ and $\bar{p} = \frac{p}{\rho\left(\frac{\nu}{h}\right)^2}$

the velocity components, after dropping bars, are given by

$u = R\,\xi\, f(\eta)$ ----- (4.4)

$v = -R\, f(\eta)$ ----- (4.5)

the stream function is given by

$\psi = -R\,\xi\, f(\eta)$ ---- (4.6)

and the pressure drop in the flow direction is given by

$[p(0,\eta) - p(\xi,\eta)] = -AR\frac{\xi^2}{2}$----- (4.7)

The shearing stress on the lower stretching sheet and the upper impermeable top are

$\tau_0 = \mu\frac{\partial u}{\partial y} = +\mu C_x \frac{1}{h} f'(0)$----- (4.8)

and $\tau_1 = -\mu\frac{\partial u}{\partial y} = -\mu C_x \frac{1}{h} f'(1)$ --- (4.9)

respectively

Thus the coefficient of skin-friction at the bottom and the top are given by

$(C\,f)_{\eta=0} = \frac{\tau_0}{?\left[\frac{v/h}{h}\right]} = R\xi f'(0)$ ----- (4.10)

$(C\,f)_{\eta=1} = \frac{\tau_1}{?\left[\frac{v/h}{h}\right]} = -R\xi f'(1)$ ----- (4.11)

Conclusion

The study shows that the steady two–dimensional laminar boundary layer non-Newtonian (power-law fluid/viscoelastic fluid) fluid flow and heat transfer over a stretching sheet under different physical situations, and the concern the effects of magnetic field, thermal radiation parameter, reaction rate parameter a suction velocity varying exponentially with time on the flow and heat transfer of a incompressible fluid over a semi-infinite vertical porous plate. The results obtained and the discussion of the results on the flow and heat transfer for the above mathematical modeling problem has been investigated during the study of above mention equations. The salient conclusionsthe coefficient of skin-friction at the bottom and the top investigated in the paper. Steady laminar flow of a viscous Incompressible fluid through a permeable bed of finite thickness h, having an impermeable top and a stretching sheet at the bottom, is considered. The fluid motion is caused by injection fluid at the top and stretching of the sheet at the bottom. A cartesian co-ordinate system is used with the origin at the bottom and y-axis normal to it.

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